Analysis of a Coulomb correction to the eikonal description of reactions

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Outline

- Introduction on halo nuclei
- Eikonal description of reactions
- Why a Coulomb correction?
- Analysis on ^{11}Be Coulomb breakup
- Conclusions and perspectives

Introduction: Halo Nuclei

Exotic nuclei with very peculiar quantum structure:

- Light, very n-rich nuclei
- Characterised by a very large matter radius
- Exhibit a low separation energy of 1 or 2 neutrons

$\Rightarrow$ strongly clusterised system:
neutrons tunnel far from the core and form a halo
Examples:

Nucleus	S_n or S_{2n}
$^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} + \mathrm{n}$	0.504 MeV
$^{11}\mathrm{Li} \equiv {}^{9}\mathrm{Li} + 2\mathrm{n}$	0.300 MeV

Breakup reaction

Halo nuclei are short-lived $\Rightarrow$ studied in indirect ways

Breakup $\equiv$ dissociation of core + halo structure
by interaction with a target

$\Rightarrow$ Need accurate theoretical description of breakup
coupled to realistic model of projectile

Various breakup models exist: CDCC, DEA, eikonal,
perturbation theory,...

- Elaborate models (CDCC, DEA) are expensive
  $\Rightarrow$ limited in projectile description (two body)
- Simpler models (eikonal, pert.) limited application

$\Rightarrow$ Seek to improve eikonal to go to three body

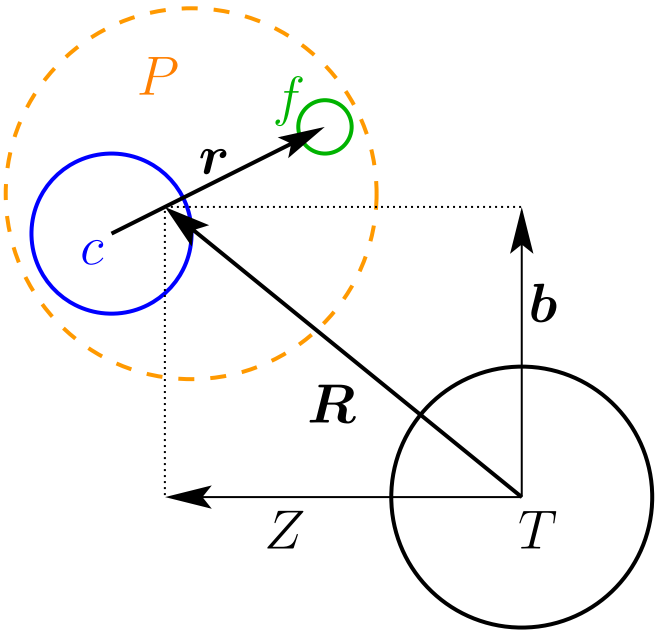
Framework

Projectile (P) modelled as a two-body system:
core (c)+loosely bound nucleon (f) described by

$$H_0 = T_r + V_{cf}(\boldsymbol{r})$$

V_{cf} adjusted to reproduce
bound states and resonances

Target T seen as
structureless particle



P-T interaction simulated by optical potentials

$\Rightarrow$ breakup reduces to three-body scattering problem:

$$[T_R + H_0 + V_{cT} + V_{fT}]\,\Psi(\boldsymbol{R},\boldsymbol{r}) = E_T\Psi(\boldsymbol{R},\boldsymbol{r})$$

Eikonal model (1)

Three-body scattering problem

$$[T_R + H_0 + V_{cT} + V_{fT}]\,\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0(\boldsymbol{r})$

To remove the rapid variation in $\boldsymbol{R}$ we factorise $\Psi(\boldsymbol{r},\boldsymbol{R}) = e^{iKZ}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{R})$:

$$\begin{aligned} H\Psi &= e^{iKZ}[T_R + vP_Z + \frac{1}{2}\mu_{PT}v^2 \\ &\quad + (H_0 + V_{cT} + V_{fT})]\widehat{\Psi} \end{aligned}$$

Neglecting T_R vs P_Z and using $E_T = \frac{1}{2}\mu_{PT}v^2 + E_0$

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - E_0 + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

Eikonal model (2)

$$i\hbar v \frac{\partial}{\partial Z} \widehat{\Psi}(\boldsymbol{r}, \boldsymbol{b}, Z) = [H_0 - E_0 + V_{cT} + V_{fT}] \widehat{\Psi}(\boldsymbol{r}, \boldsymbol{b}, Z)$$

This is the Dynamical Eikonal Approximation
[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

Usual eikonal makes adiabatic approx. $H_0 - E_0 \sim 0$
$\Rightarrow$ neglects dynamical effects

$$\widehat{\Psi}^{\text{eik}}(\boldsymbol{r}, \boldsymbol{b}, Z \to \infty) = e^{i\chi(\boldsymbol{r}, b)} \Phi_0(\boldsymbol{r}),$$

$$\chi(\boldsymbol{r}, b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} dZ \, [V_{cT}(\boldsymbol{r}, \boldsymbol{b}, Z) + V_{fT}(\boldsymbol{r}, \boldsymbol{b}, Z)]$$

DEA improves eikonal including dynamical effects
BUT DEA is expensive
eikonal cannot handle Coulomb interaction
$\Rightarrow$ can we correct Coulomb in eikonal?

Coulomb Corrected Eikonal

The eikonal Coulomb phase reads

$$\chi_C(\boldsymbol{r}, b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} \frac{Z_c Z_T e^2}{R_{cT}} dZ \propto \frac{1}{b}$$

$\Rightarrow e^{i\chi_C} = 1 + i\chi_C - \frac{1}{2}\chi_C^2 + \ldots$ diverges when $\int db$

Idea: replace χ_C by χ_{FO} from perturbation theory
[Margueron, Bonaccorso, and Brink, NPA 720, 337 (2003)]

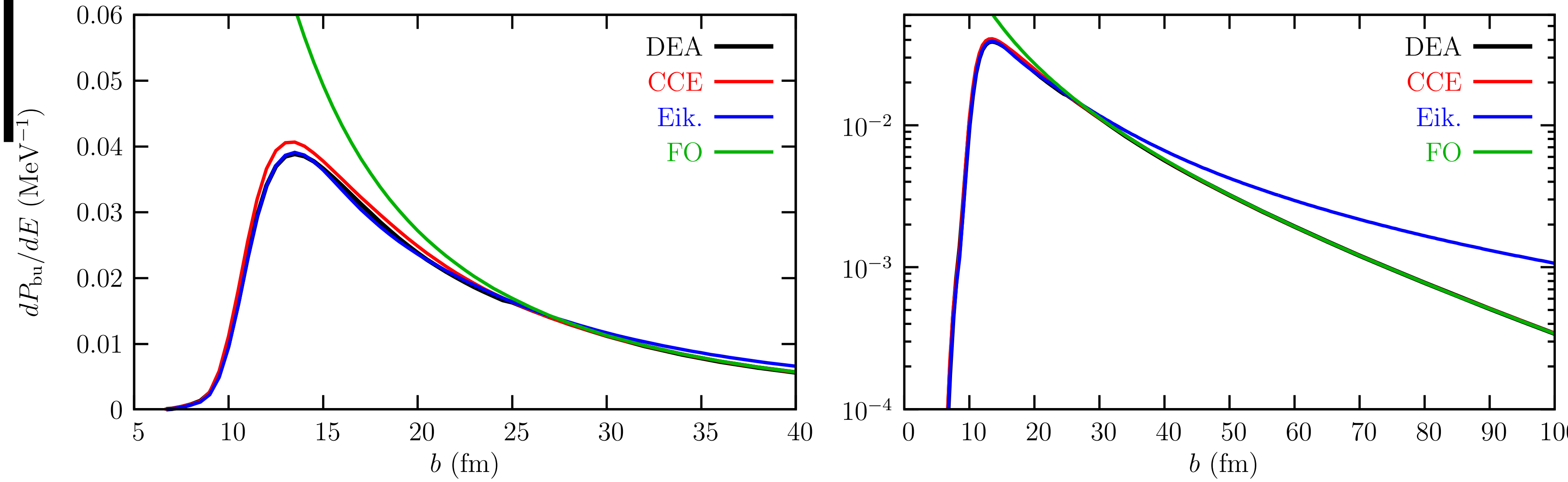
$$\chi_{FO}(\boldsymbol{r}, b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} e^{i\omega Z}\frac{Z_c Z_T e^2}{R_{cT}} dZ \propto e^{-\omega b},$$

with correct asymptotics. The correction then reads

$$e^{i\chi} = e^{i\chi_N}\left(e^{i\chi_C} - i\chi_C + i\chi_{FO}\right)$$

We compare CCE with DEA on ^{11}Be breakup

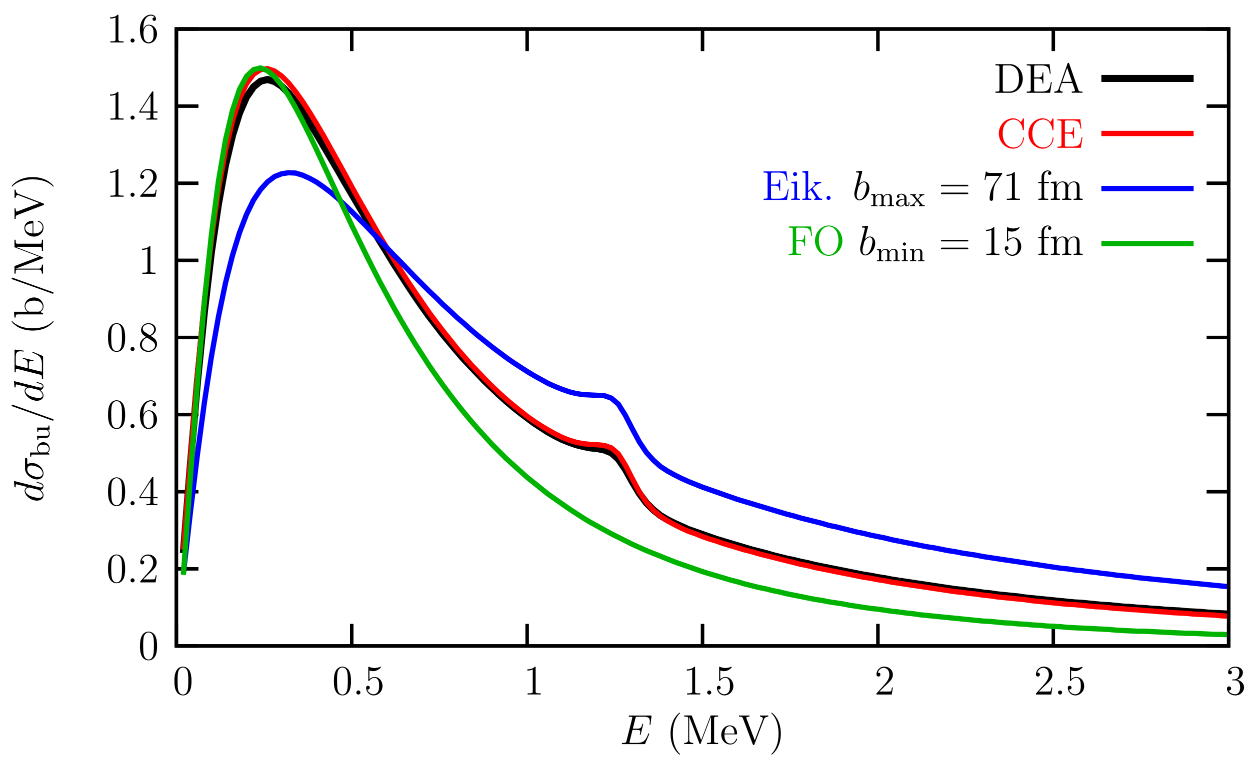
^{11}Be breakup on Pb @ $69A$MeV



- Good agreement between CCE and DEA at all b
- Eikonal ok at small b (nuclear) but $\propto 1/b$ at large b
- FO good asymptotic (Coulomb), but no nuclear

$\Rightarrow$ CCE combines advantages of eikonal and FO

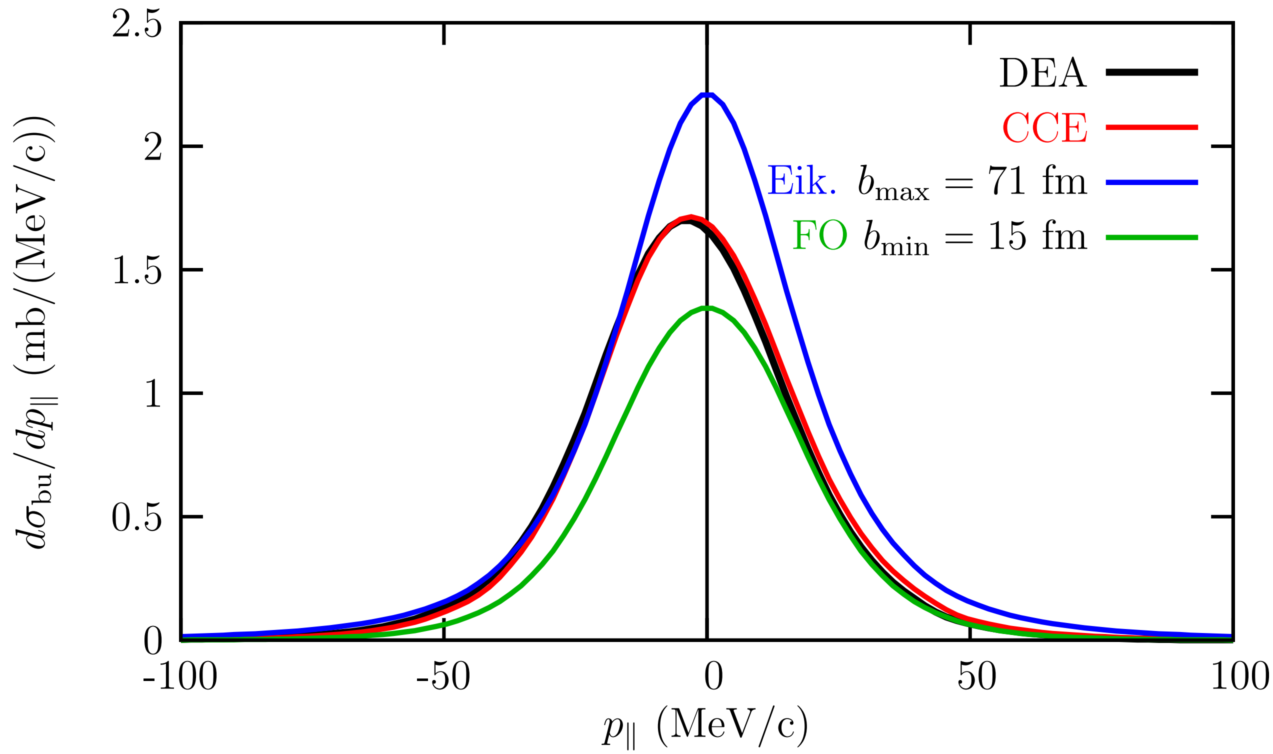
Energy distribution



- Excellent agreement between CCE and DEA
- Eikonal needs cutoff at large b; wrong shape
- FO needs cutoff at small b; lacks nuclear

$\Rightarrow$ Confirms the validity of the Coulomb correction

Momentum distribution



- Excellent agreement between CCE and DEA in particular asymmetry (dynamical effects)
- Eikonal too high and symmetric
- FO too low and symmetric

$\Rightarrow$ Coulomb correction restores dynamical effects

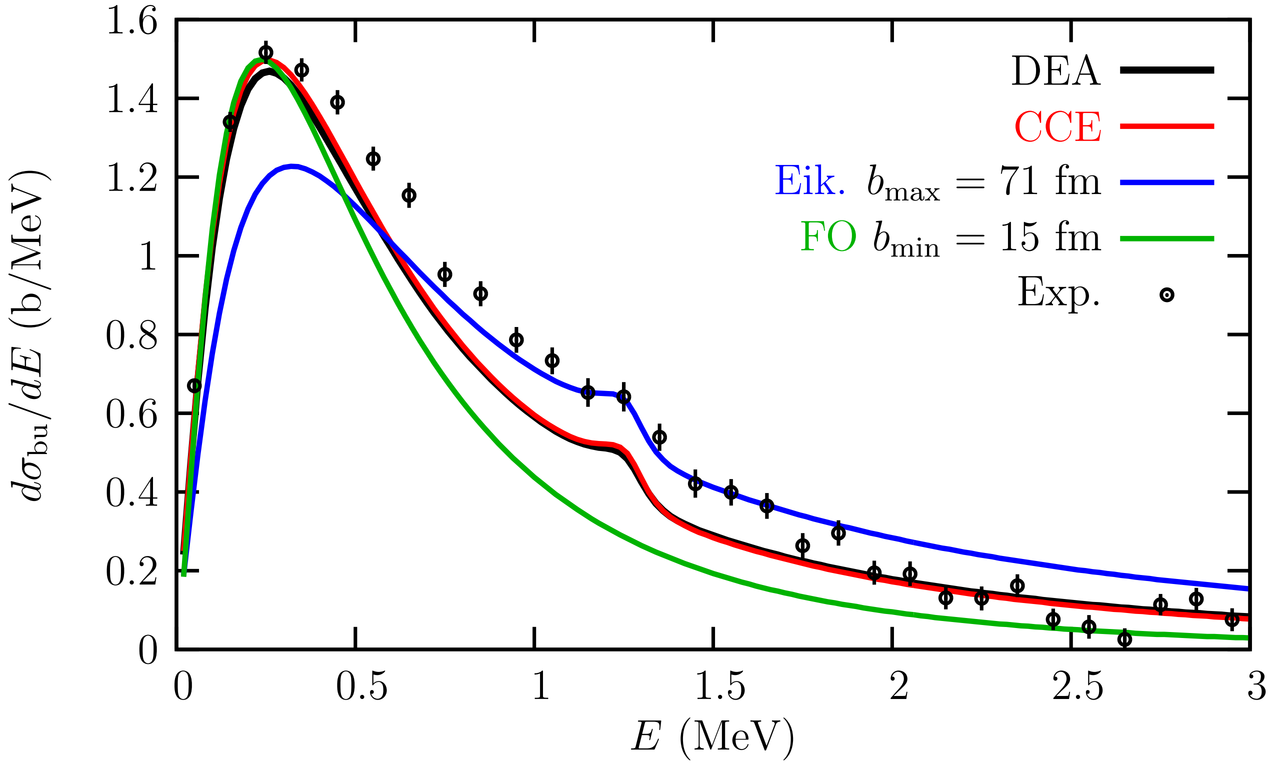
Conclusions and Perspectives

Coulomb Corrected Eikonal is a reaction model
based on eikonal model
with a First-Order correction for Coulomb interaction

- CCE corrects the wrong asymptotics of eikonal Coulomb phase
- CCE introduces nuclear interaction in FO
- CCE restores missing dynamics
- CCE in excellent agreement with DEA
- CCE 100 times faster than DEA

$\Rightarrow$ CCE to study breakup more complex projectiles
(2n halo (^{11}Li), better descriptions,...)

Including data points



Data from N. Fukuda et al. PRC 70, 054606 (2004)

Cross sections

We define breakup amplitudes

$$S_{klm}(b) = \left\langle \phi_{klm} \left| e^{i\chi^N} \left(e^{i\chi^C} - i\chi^C + i\chi^{FO} \right) \right| \phi_0 \right\rangle$$

The energy distribution reads

$$\frac{d\sigma_{\mathrm{bu}}}{dE} = \frac{4\mu_{cf}}{\hbar^2 k} \sum_{lm} \int_0^\infty b db |S_{klm}(b)|^2$$

The parallel-momentum distribution reads

$$\frac{d\sigma_{\mathrm{bu}}}{dk_{\|}} = 8\pi \int_0^\infty b db \int_{|k_{\|}|}^\infty \frac{dk}{k} \sum_m \left| \sum_l Y_l^m(\theta_k, 0) S_{klm}(b) \right|^2$$